Estimation of physical conditions in PDRs
Bayesian approach with spatial regularization

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Aims and contributions

New large hyper-spectral surveys in radio-astronomy: game changer for study of star formation, feedback mechanisms and chemistry of interstellar gas. $\Longrightarrow$ now possible to observe full molecular clouds (10 pc size) at dense core scale (<0.1 pc) spatial resolution. For instance, the IRAM-30m Large Program «Orion B» [1] provides a 10^6 pixel observation map, covering ~250 pc^2 of the Orion B cloud with emission of dozens of molecules.

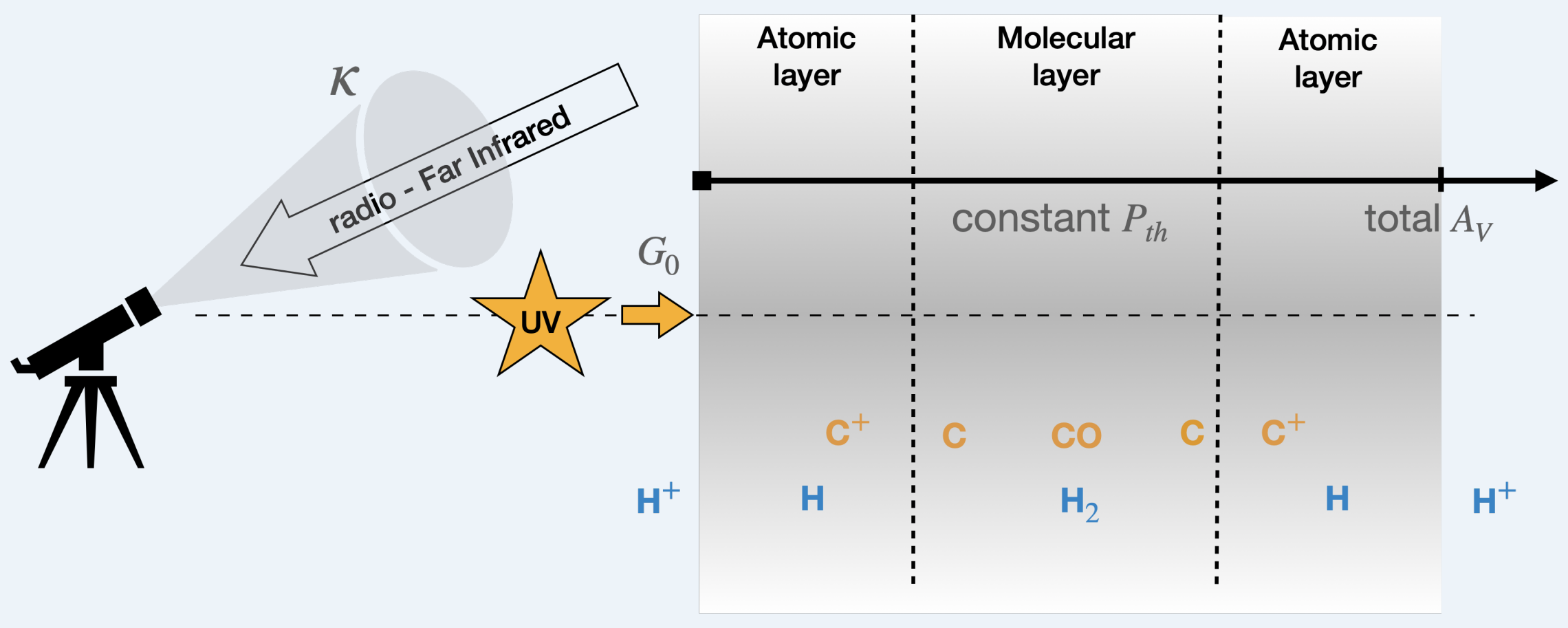
Goal To estimate maps of thermal pressure P_{th}, UV radiation intensity G_0, visual extinction A_V, scaling factor κ and the associated uncertainties.

Challenge Variability of the SNR: the brightest regions can be well constrained, but the regions with low SNR lead to degenerate solutions.

Approach
★ Mixture of both additive and multiplicative noises + limited detectability $\Longrightarrow$ more realistic observation model
★ Spatial regularization $\Longrightarrow$ exploitation of the information contained in neighbouring pixels.
★ Bayesian approach $\Longrightarrow$ access to accurate credibility intervals.

Problem formulation

➔ Meudon PDR code [2]: solves radiative transfer, chemistry, thermal balance on stationary 1D plane-parallel model.



➔ Assumption: physics in PDR = interpolation of a grid of simulations of Meudon PDR code. Defines function f s.t. for one pixel n:

$$\boxed{f : \underbrace{x_n = (\kappa, P_{th}, G_0, A_V)}_{\substack{\text{physical parameters}\\ \text{(to estimate)}}} \mapsto \underbrace{(y_{n,\ell})_{\ell=1}^{L}}_{\substack{\text{integrated intensities}\\ \text{(observed)}}}}$$

➔ observation model involves detectability limit ω, gaussian noise $\varepsilon_{n,\ell}^{(a)}$ and lognormal noise $\varepsilon_{n,\ell}^{(m)}$

$$\boxed{y_{n,\ell} = \begin{cases} \omega & \text{if } \varepsilon_{n,\ell}^{(m)} f_\ell(x_n) + \varepsilon_{n,\ell}^{(a)} \leq \omega \\ \varepsilon_{n,\ell}^{(m)} f_\ell(x_n) + \varepsilon_{n,\ell}^{(a)} & \text{else} \end{cases}}$$

$\Longrightarrow$ defines a **likelihood** function $\pi(y \mid x)$.

Bayesian approach

➔ State-of-the-art in millimeter/IR astronomy estimations: Maximum Likelihood Estimates (MLE) $\Longrightarrow$ very sensitive to noise

➔ For robust estimators: spatial regularization **prior** $\pi(x)$ (norm of image gradient or laplacian)

➔ **Posterior** probability density function:

$$\boxed{\pi(x \mid y) \propto \pi(y \mid x)\pi(x)}$$

➔ To derive estimators and credibility intervals from posterior distribution: **sample** from it with Monte Carlo Markov Chain (MCMC).

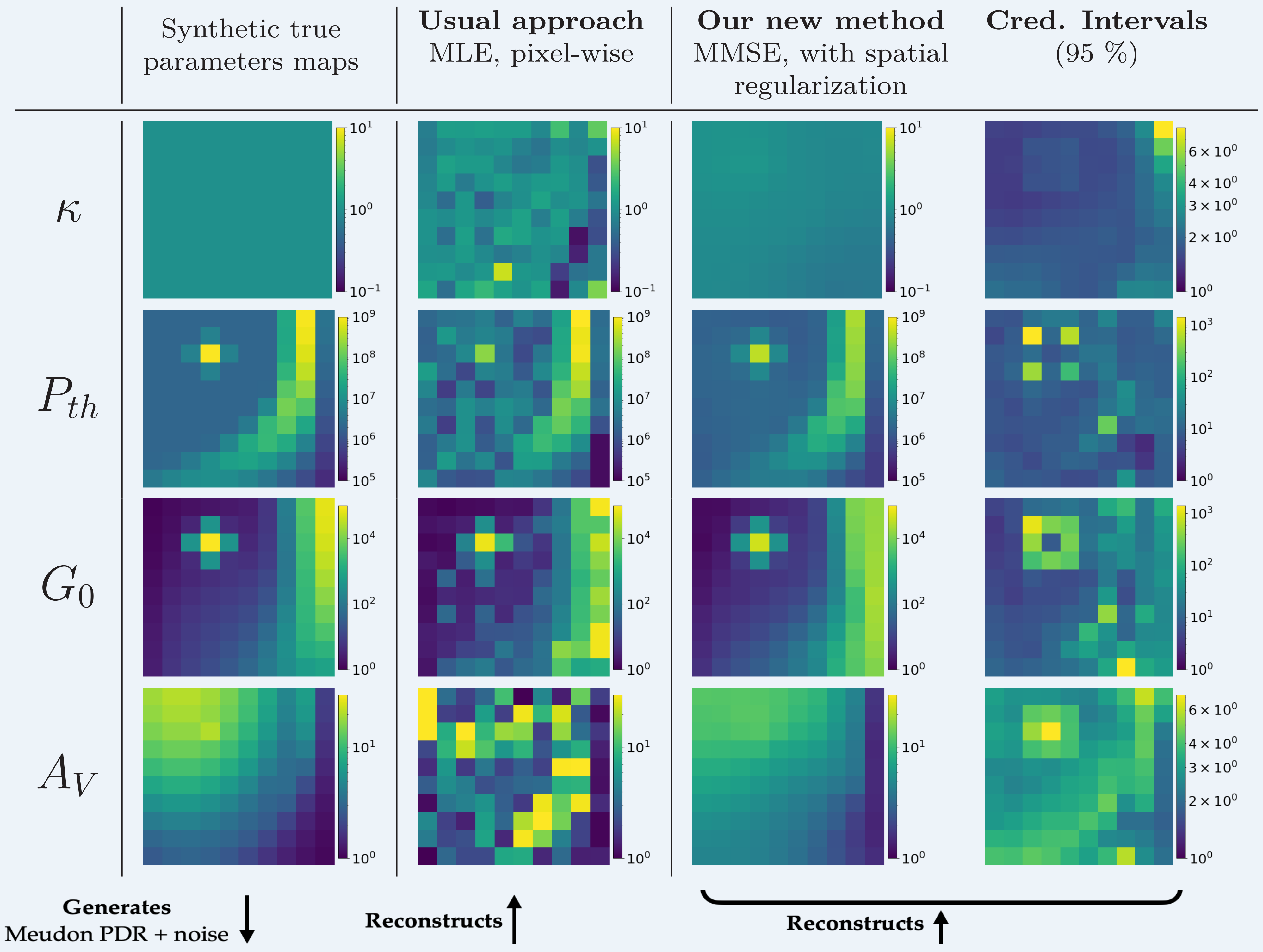
➔ Our sampler mixes two kernels: one identifies local minima, one efficiently explores them.

➔ Our ponctual estimator: MMSE (Mininimum Mean Squared Error) = mean of posterior.

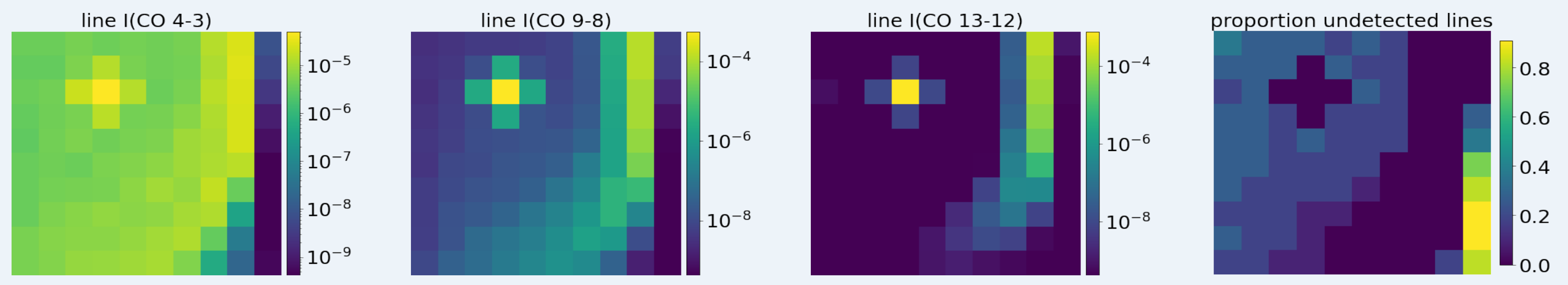
References

[1] Pety *et al.*, *The anatomy of the Orion B Giant Molecular Cloud: A local template for studies of nearby galaxies*, 2016
[2] Le Petit *et al.*, *A model for atomic and molecular interstellar gas the Meudon PDR code*, 2009
[3] Joblin *et al.*, *Structure of photodissociation fronts in star-forming regions revealed by Herschel observations of high-J CO emission lines*, 2018

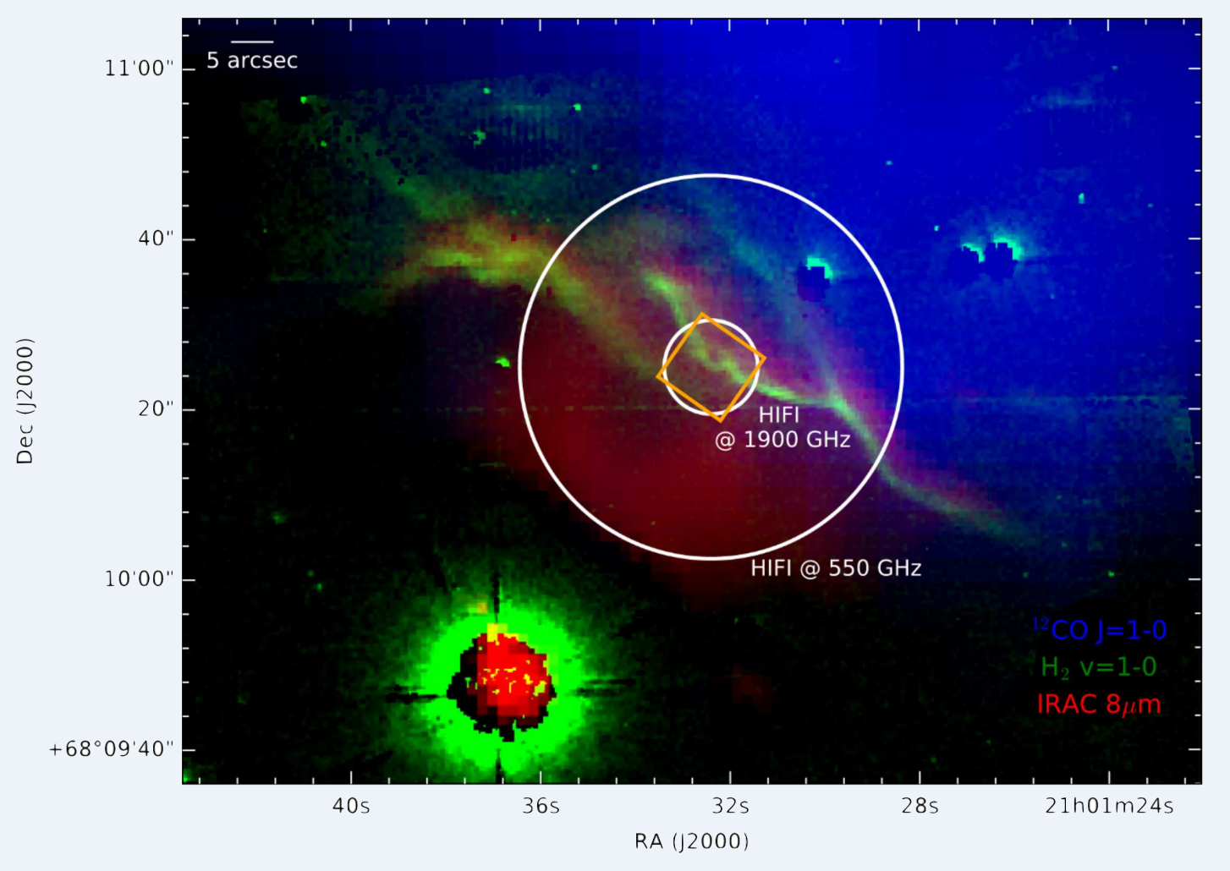
Application on toy dataset (100 pixels)

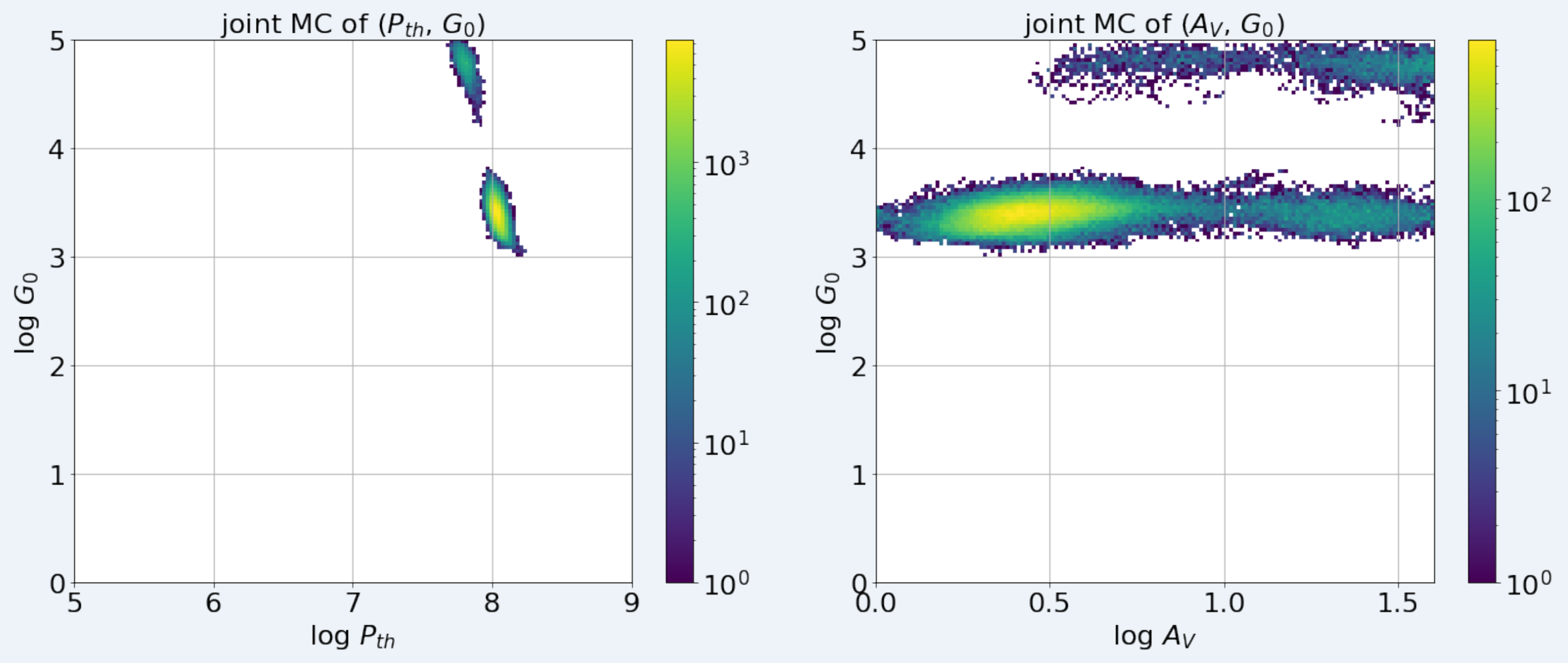


Synthetic observations: 10 excited lines of CO (from J=4-3 to 13-12)



NGC 7023 (1 pixel)





➔ Estimation from 10 lines of CO and 2 lines of CI.

➔ In contrast with [3], G_0 and A_V estimated along with κ and P_{th}.

➔ Two regions of acceptable solutions identified.

Summary for main mode

	MLE [3]	MMSE	2.5%	97.5%
κ	0.7	4.8	2.5	8.6
P_{th} ($\times 10^8$)	1	1.1	1.0	1.3
G_0 ($\times 10^3$)	2.6	1.6	1.0	2.6
A_V	10	3.1	1.6	30.5

Conclusions

Achieved:
✓ Spatial regularization $\Longrightarrow$ more robust estimations of maps of parameters,
✓ Bayesian approach $\Longrightarrow$ more complete description than ponctual estimators.

Future Work:
➛ Application to other environments than PDR (shocks, dark clouds, etc.),
➛ Code acceleration to scale to ~ 10^6 pixel observations (e.g., Orion B IRAM Large Programm).