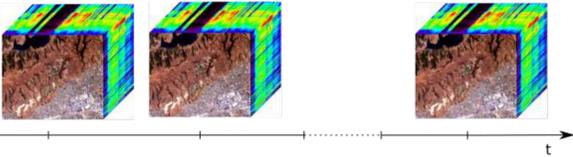


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1. Introduction

Hyperspectral imagery

- high spectral resolution, low spatial resolution \Rightarrow hyperspectral unmixing
- hyperspectral unmixing
 - \triangleright identifying the reference spectral signatures in the data (*endmembers*)
 - \triangleright estimating the endmember relative fraction in each pixel (*abundances*).



Unmixing multi-temporal hyperspectral images

- T hyperspectral images acquired over the same area
- varying acquisition conditions + inherent variability of the imaged scene (natural evolution) \Rightarrow *variability*
- *increasing number of available images* (several large images, significant number of images)
- \triangleright *online estimation* (sequential analysis)

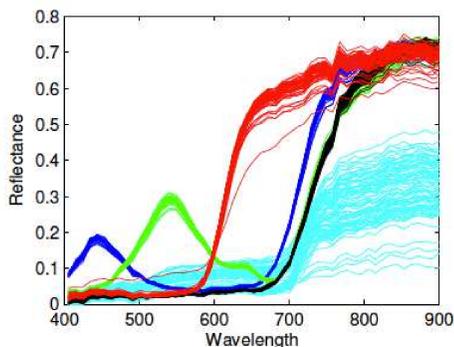


Figure 1: Spectral variability (P. Gader, A. Zare, R. Close, J. Aitken, G. Tuell, MUUFL Gulfport Hyperspectral and LiDAR Airborne Data Set, University of Florida, Gainesville, FL, Tech. Rep. REP-2013-570, Oct. 2013.)

2. Model

Assumptions

- \triangleright the T images of the sequence share K endmembers (K known)
- \triangleright the pixels of *each image* are similarly affected by spectral variability (first approximation).

Perturbed linear mixing model (PLMM)

- pixel spectrum = *linear combination of corrupted endmembers*
- corrupted endmembers = endmembers affected by an *additive time-varying perturbation vector*

$$\mathbf{y}_{nt} = \sum_{k=1}^K a_{knt} (\mathbf{m}_k + \mathbf{d}\mathbf{m}_{kt}) + \mathbf{b}_{nt} \quad (1)$$

Matrix formulation

$$\mathbf{Y}_t = (\mathbf{M} + \mathbf{d}\mathbf{M}_t) \mathbf{A}_t + \mathbf{B}_t \quad (2)$$

N number of pixels
 L number of spectral bands
 K number of endmembers
 $\mathbf{Y}_t = [\mathbf{y}_{1t}, \dots, \mathbf{y}_{Nt}] \in \mathbb{R}^{L \times N}$ t th hyperspectral image
 $\mathbf{M} = [\mathbf{m}_1, \dots, \mathbf{m}_K] \in \mathbb{R}^{L \times K}$ endmember matrix
 $\mathbf{A}_t = [\mathbf{a}_{1t}, \dots, \mathbf{a}_{Nt}] \in \mathbb{R}^{K \times N}$ t th abundance matrix
 $\mathbf{d}\mathbf{M}_t = [\mathbf{d}\mathbf{m}_{1t}, \dots, \mathbf{d}\mathbf{m}_{Kt}] \in \mathbb{R}^{L \times K}$ t th variability matrix

Constraints

- *abundance and endmembers* (physical considerations)

$$\mathbf{M} \succeq \mathbf{0}_{L,K}, \mathbf{A}_t \succeq \mathbf{0}_{K,N}, \mathbf{A}_t^T \mathbf{1}_K = \mathbf{1}_N, \forall t \in \{1, \dots, T\} \quad (3)$$

- *variability* (modeling): small average temporal variability + upper bound for the instantaneous variability energy

$$\left\| \frac{1}{T} \sum_{t=1}^T \mathbf{d}\mathbf{M}_t \right\|_F \leq \kappa, \quad \|\mathbf{d}\mathbf{M}_t\|_F \leq \sigma, \quad \forall t \in \{1, \dots, T\} \quad (4)$$

3. Problem formulation

- Two-stage stochastic problem, associated with the empirical risk minimization

$$\min_{\mathbf{M} \in \mathcal{M}} \frac{1}{T} \sum_{t=1}^T h(\mathbf{Y}_t, \mathbf{M}) + \beta \Psi(\mathbf{M}) \quad (5)$$

$$h(\mathbf{Y}_t, \mathbf{M}) = \min_{(\mathbf{A}, \mathbf{d}\mathbf{M}) \in \mathcal{A}_K \times \mathcal{D}_t} f(\mathbf{Y}_t, \mathbf{M}, \mathbf{A}, \mathbf{d}\mathbf{M}) \quad (6)$$

- $\triangleright f$: regularized *instantaneous discrepancy measure*
- $\triangleright h$: *cost of the t th optimal decision* to update the endmember matrix \mathbf{M} given the *data available at time t*
- $\triangleright \Psi$: endmember regularization.
- $\triangleright \mathcal{M} = \{\mathbf{M} : \mathbf{M} \succeq \mathbf{0}_{L,K}\}$
- $\triangleright \mathcal{A}_K = \{\mathbf{A} : \mathbf{A} \succeq \mathbf{0}_{K,N}, \mathbf{A}^T \mathbf{1}_K = \mathbf{1}_N\}$
- $\triangleright \mathcal{D}_t = \{\mathbf{d}\mathbf{M} : \|\mathbf{d}\mathbf{M}\|_F \leq \sigma\} \cap \{\mathbf{d}\mathbf{M} : \|\mathbf{d}\mathbf{M} + \mathbf{E}_{t-1}\|_F \leq t\kappa\}$
- $\triangleright \mathbf{E}_t = \sum_{i=1}^t \mathbf{d}\mathbf{M}_i$.

- *White Gaussian noise assumption*

$$f(\mathbf{Y}_t, \mathbf{M}, \mathbf{A}, \mathbf{d}\mathbf{M}) = \frac{1}{2} \|\mathbf{Y}_t - (\mathbf{M} + \mathbf{d}\mathbf{M})\mathbf{A}\|_F^2 + \alpha \Phi_t(\mathbf{A}) + \gamma \Upsilon_t(\mathbf{d}\mathbf{M}) \quad (7)$$

- $\triangleright \Phi_t, \Upsilon_t$: appropriate regularizations
- \triangleright trade-off between the data fitting term and the penalties $\Phi_t(\mathbf{A})$, $\Psi(\mathbf{M})$ and $\Upsilon_t(\mathbf{d}\mathbf{M})$ controlled by (α, β, γ) .

Abundance and variability regularization

Moderate/smooth changes assumed from one image to another

$$\Phi_t(\mathbf{A}) = \frac{1}{2} \|\mathbf{A} - \mathbf{A}_{t-1}\|_F^2 \quad (8)$$

$$\Upsilon_t(\mathbf{d}\mathbf{M}) = \frac{1}{2} \|\mathbf{d}\mathbf{M} - \mathbf{d}\mathbf{M}_{t-1}\|_F^2 \quad (9)$$

Endmember regularization

Constrains the volume of the simplex whose vertices are the endmember signatures

$$\Psi(\mathbf{M}) = \frac{1}{2} \sum_{i=1}^K \left(\sum_{j=1, j \neq i}^K \|\mathbf{m}_i - \mathbf{m}_j\|_2^2 \right). \quad (10)$$

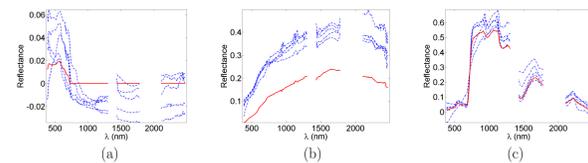


Figure 2: Example of endmembers (in red) and variability (in blue) obtained when removing the constraint on the averaged variability.

4. An online algorithm

Structure of the online algorithm

- whenever an image \mathbf{Y}_t is received, *local abundance and variability estimation* by a proximal alternating linearized minimization (PALM) algorithm
- \triangleright PALM *guaranteed to converge to a critical point of the non-convex problem (6)*

- endmembers updated by proximal gradient descent steps

- possibility to add a forgetting factor $\xi \in]0, 1]$

- provided problem (6) exclusively admits locally unique critical points, *Algo. 1 converges to a critical point of Problem (5) as $T \rightarrow +\infty$.*

Algorithm 1: Online unmixing algorithm.

```

Data:  $\mathbf{M}_0, \mathbf{A}_0, \mathbf{d}\mathbf{M}_0, \alpha > 0, \beta > 0, \gamma > 0, \xi \in ]0, 1]$ 
begin
   $\mathbf{C}_0 \leftarrow \mathbf{0}_{K,K}$ ;
   $\mathbf{D}_0 \leftarrow \mathbf{0}_{L,K}$ ;
   $\mathbf{E}_0 \leftarrow \mathbf{0}_{L,K}$ ;
  for  $t = 1$  to  $T$  do
    a Random selection of an image  $\mathbf{Y}_t$ ;
    // Abundance and variability estimation by PALM
    b  $(\mathbf{A}_t, \mathbf{d}\mathbf{M}_t) \in \arg \min_{(\mathbf{A}, \mathbf{d}\mathbf{M}) \in \mathcal{A}_K \times \mathcal{D}_t} f(\mathbf{Y}_t, \mathbf{M}_t, \mathbf{A}, \mathbf{d}\mathbf{M})$ ;
     $\mathbf{C}_t \leftarrow \xi \mathbf{C}_{t-1} + \mathbf{A}_t \mathbf{A}_t^T$ ;
     $\mathbf{D}_t \leftarrow \xi \mathbf{D}_{t-1} + (\mathbf{d}\mathbf{M}_t \mathbf{A}_t - \mathbf{Y}_t) \mathbf{A}_t^T$ ;
     $\mathbf{E}_t \leftarrow \xi \mathbf{E}_{t-1} + \mathbf{d}\mathbf{M}_t$ ;
    // Endmember update
    c  $\mathbf{M}_t \leftarrow \arg \min_{\mathbf{M} \in \mathcal{M}} \frac{1}{2} [\frac{1}{2} \text{Tr}(\mathbf{M}^T \mathbf{C}_t) + \text{Tr}(\mathbf{M}^T \mathbf{D}_t)] + \beta \Psi(\mathbf{M})$ 
  end
Result:  $\mathbf{M}_T, (\mathbf{A}_t)_{t=1, \dots, T}, (\mathbf{d}\mathbf{M}_t)_{t=1, \dots, T}$ 
    
```

5. Experiment with synthetic data

- Method evaluated on 15 linear mixtures of size 31×30 , composed of 413 bands
- No pure pixel, mixtures corrupted by an additive white Gaussian noise to ensure SNR = 30 dB
- *Abundance and endmembers initialized with VCA/FCLS*
- Simulation scenario: Algo. 1 run for 50 cycles through the whole dataset, PALM and proximal gradient descent stopped after 50 iterations, $\xi = 0.99$, $\alpha = 3.9 \times 10^{-2}$, $\beta = 5.4 \times 10^{-4}$, $\gamma = 3.2 \times 10^{-4}$, $\sigma^2 = 12.4$, $\kappa^2 = 1.9$.

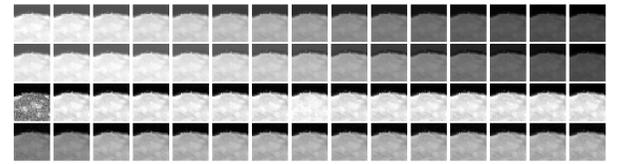


Figure 3: Abundance maps of \mathbf{m}_{1t} [0: black, 1: white] (rows: true maps, proposed method, VCA/FCLS, SISAL/FCLS).

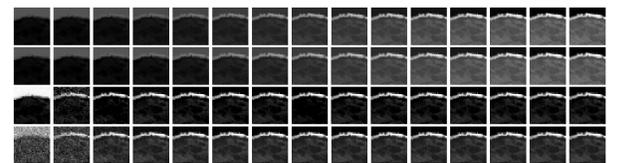


Figure 4: Abundance maps of \mathbf{m}_{2t} [0: black, 1: white] (rows: true maps, proposed method, VCA/FCLS, SISAL/FCLS).

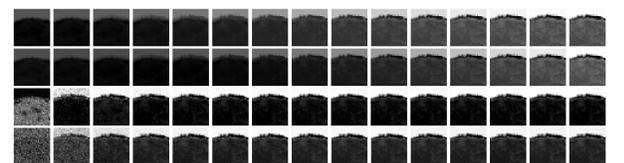


Figure 5: Abundance maps of \mathbf{m}_{3t} [0: black, 1: white] (rows: true maps, proposed method, VCA/FCLS, SISAL/FCLS).

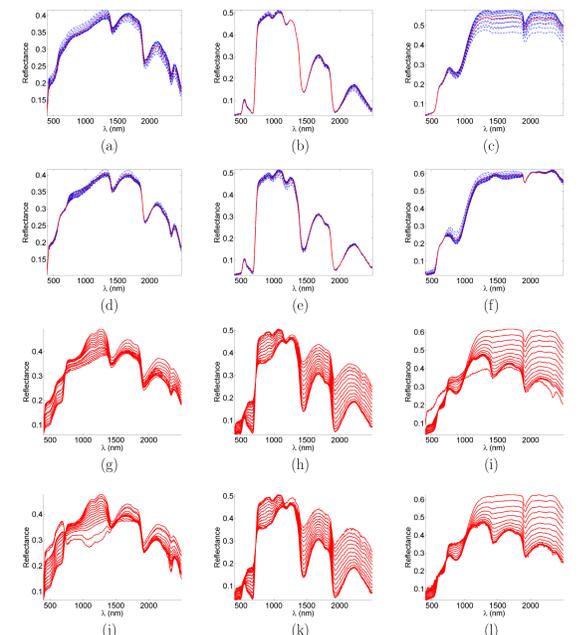


Figure 6: Corresponding endmembers [rows: true endmembers (in red) and variability (in blue), proposed method, VCA, SISAL].

Table 1: Simulation results obtained with synthetic data ($\text{GMSE}(\mathbf{A}) \times 10^{-2}$, $\text{GMSE}(\mathbf{d}\mathbf{M}) \times 10^{-4}$, $\text{RE} \times 10^{-5}$).

	VCA/FCLS	SISAL/FCLS	Prop. method
aSAM(M) (°)	8.9792	8.6685	1.9898
GMSE(A)	6.67	3.90	0.47
GMSE(dM)	/	/	3.07
RE	9.59	9.49	9.63
time (s)	2	2.2	561

6. Conclusion and future work

- Proposition of an *online hyperspectral unmixing algorithm accounting for endmember temporal variability*
- \triangleright Consider *abrupt endmember changes* (common in real data)
- \triangleright Incorporate spatial variability
- \triangleright Find automatic rules to adjust the regularization parameters