A Faceted Prior for Scalable Wideband Computational Imaging

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1. Context and proposed approach

- **Objective:** form extreme size wideband image $\mathbf{X}$ from incomplete data

$$\begin{bmatrix} \mathbf{Y} \end{bmatrix} = \boldsymbol{\Phi} \left(\begin{bmatrix} \mathbf{X} \end{bmatrix} \right) + \mathbf{N}, \ \mathbf{X} \in \mathbb{R}^{N \times L}, \ \mathbf{Y}, \mathbf{N} \in \mathbb{C}^{M \times L}$$

$\rightsquigarrow$ optimization problem: $\underset{\mathbf{X} \in \mathbb{R}_+^{N \times L}}{\text{minimize}} \ \underbrace{f\big(\mathbf{Y}, \boldsymbol{\Phi}(\mathbf{X})\big)}_{\text{data fitting}} + \underbrace{r(\mathbf{X})}_{\text{regularization}}$

- Spectral correlations: low-rankness and joint-sparsity regularization

$$r(\mathbf{X}) = \lambda \|\mathbf{X}\|_* + \mu \|\boldsymbol{\Psi}^\dagger \mathbf{X}\|_{2,1}, \ \boldsymbol{\Psi}^\dagger \in \mathbb{R}^{P \times N} \text{ sparsifying dictionary}$$

$\rightsquigarrow$ prohibitive cost: radio-astronomy, $L \approx 10^3$ channels, $N \approx 10^{14}$ pixels

- **Proposed approach:** facet-based low-rankness and joint sparsity prior

$$r(\mathbf{X}) = \sum_{i=1}^{I} \lambda_i \|\mathbf{W}_i \tilde{\mathbf{S}}_i \mathbf{X}\|_* + \mu_i \|\boldsymbol{\Psi}_i^\dagger \mathbf{S}_i \mathbf{X}\|_{2,1}$$

  - $\rightsquigarrow$ define dictionary $\boldsymbol{\Psi}_i^\dagger$ based on the structure of $\boldsymbol{\Psi}^\dagger$; (exact decomposition when $\boldsymbol{\Psi}^\dagger$ is a wavelet dictionary [1])
  - $\rightsquigarrow$ more scalable, promotes local spectral correlations;
  - $\rightsquigarrow$ $\mathbf{S}_i, \tilde{\mathbf{S}}_i$ selection operators, weights $\mathbf{W}_i$ to mitigate faceting artifacts.

2. Application to radio-astronomy

- **Measurement operator:**

$$\boldsymbol{\Phi}(\mathbf{X}) = \{\boldsymbol{\Phi}_\ell \mathbf{x}_\ell\}_{\ell=1}^{L}, \quad \boldsymbol{\Phi}_\ell = \boldsymbol{\Theta}_\ell \mathbf{G}_\ell \mathbf{F} \mathbf{Z}$$

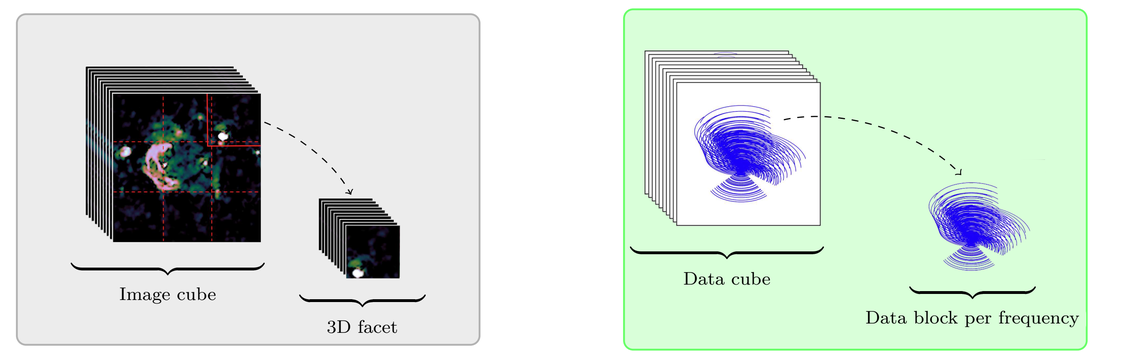
$\mathbf{x}_\ell \in \mathbb{R}^N$	image in channel ℓ (column of $\mathbf{X}$)
$\mathbf{F} \in \mathbb{C}^{K \times K}$	Fourier transform
$\mathbf{Z} \in \mathbb{R}^{K \times N}$	zero-padding and scaling operator
$\boldsymbol{\Theta}_\ell \in \mathbb{R}^{M \times M}$	natural weighting matrix
$\mathbf{G}_\ell \in \mathbb{C}^{M \times K}$	interpolation and calibration kernels

- **Problem formulation:** extension of HyperSARA [2] – a wideband radio-interferometric (RI) imaging approach

  $\rightsquigarrow$ data fitting term: per channel ℓ_2 constraint, controlled by ε_ℓ

$$\underset{\mathbf{X} \in \mathbb{R}_+^{N \times L}}{\text{minimize}} \ \underbrace{\sum_{\ell=1}^{L} \iota_{\mathcal{B}(\mathbf{y}_\ell, \varepsilon_\ell)}\big(\boldsymbol{\Phi}_\ell \mathbf{x}_\ell\big)}_{f(\mathbf{Y}, \boldsymbol{\Phi}(\mathbf{X}))} + \underbrace{\sum_{i=1}^{I} \|\mathbf{W}_i \tilde{\mathbf{S}}_i \mathbf{X}\|_* + \mu_i \|\boldsymbol{\Psi}_i^\dagger \mathbf{S}_i \mathbf{X}\|_{2,1}}_{r(\mathbf{X})}$$

- **Imaging algorithm:** preconditioned primal-dual algorithm [3].



3. Illustration on synthetic data

- **Simulation settings:**
  - synthetic wideband image of the W28 supernova remnant;
  - $L = 20$ spectral channels, $N = 1024 \times 1024$ pixels;
  - $M \approx 0.5N$ measurements per channel, SNR = 60 dB;
  - faceted HyperSARA compared to HyperSARA [2] and single channel reconstruction (SARA [4]):
    - ▷ quality comparable to HyperSARA, much lower computing time;
    - ▷ overall reconstruction improvement of very low intensity emissions;
    - ▷ limited performance of SARA: single channel $\rightsquigarrow$ limited range of spatial frequencies exploited (nature of RI Fourier sampling).

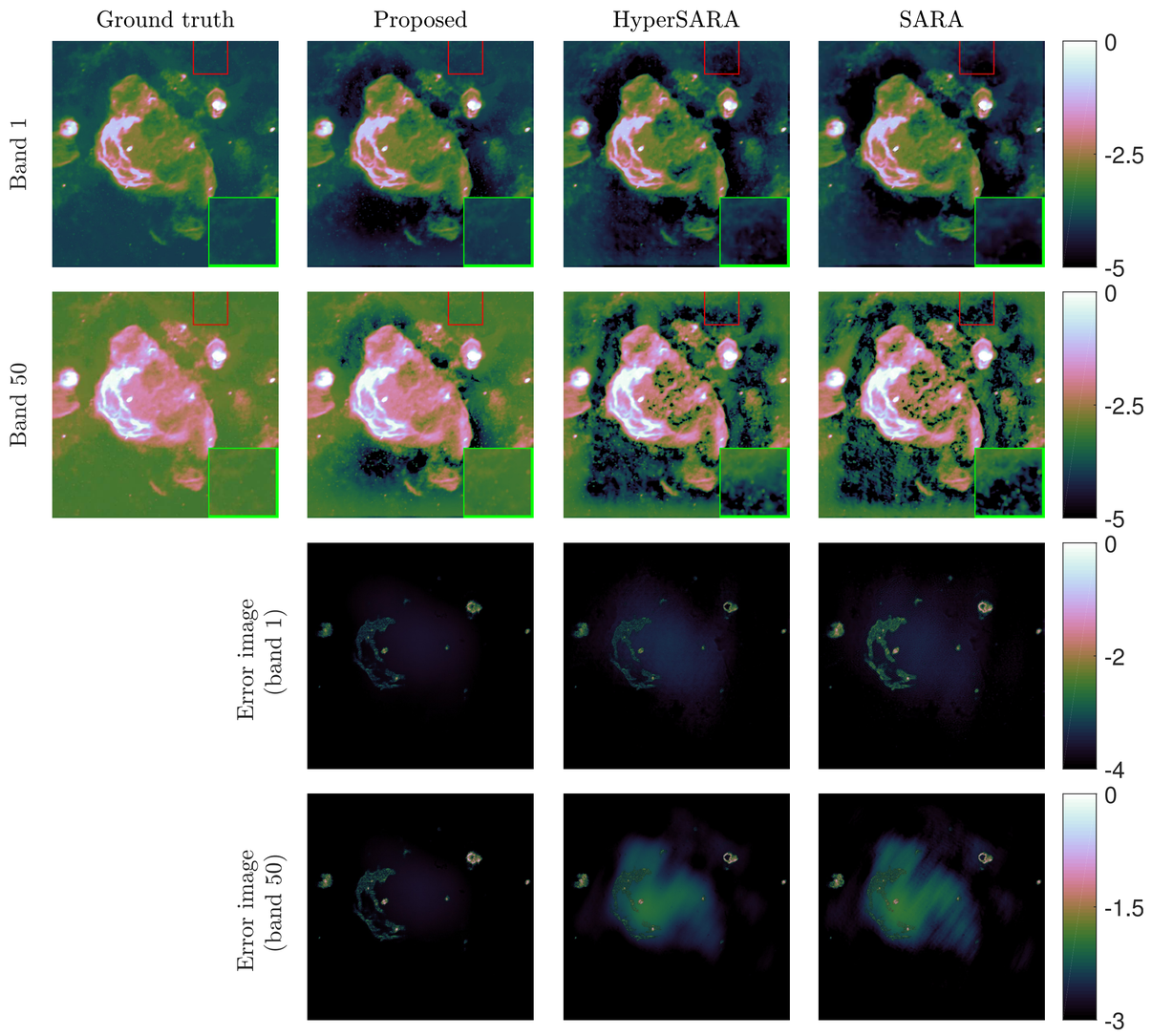


Figure 1: Reconstructed and error images (in rows) displayed in log scale for the different approaches. The reconstruction SNR for the displayed channels and per iteration computing time are: (proposed) [36.97/36.97 dB, 7.8 s], (HyperSARA) [SNR = 38.83/37.83 dB, 45 s], (SARA) [27.78/37.06 dB, 2.2 s].

4. Conclusions and perspectives

- **Conclusions:**
  - ✓ faceted prior for scalable wideband imaging;
  - ✓ promote local spectral correlations *via* a facet-based nuclear norm
    - $\rightsquigarrow$ improved reconstruction of faint emissions in RI, compared to HyperSARA.
- **Perspectives:**
  - → control number of facets handled at each iteration: randomization;
  - → investigate faceting of Fourier transform involved in RI measurement operator: preserve both data and image locality.

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